Mathematics: analysis and approaches HL

Timezone 1

To protect the integrity of the assessments, increasing use is being made of examination variants. By using variants of the same examination, students in one part of the world will not always be responding to the same examination content as students in other parts of the world. A rigorous process is applied to ensure that the content across all variants is comparable in terms of difficulty and syllabus coverage. In addition, measures are taken during the standardisation and grade awarding processes to ensure that the final grade awarded to students is comparable.

Contents

Grade boundaries

Higher level overall

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 22	23 - 34	35 - 48	49 - 61	62 - 75	76 - 100

Higher level internal assessment

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

Higher level paper one

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 23	24 - 34	35 - 49	50 - 63	64 - 79	80 - 110

Higher level paper two

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 22	23 - 33	34 - 48	49 - 62	63 - 78	79 - 110

Higher level paper three

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 5	6 - 14	15 - 22	23 - 30	31 - 36	37 - 45	46 - 55

Higher level internal assessment

The range and suitability of the work submitted

The concern that several teachers have not read the subject reports persists. It is unfortunate and unfair to students when teachers upload explorations which are not annotated which makes it difficult for the moderator to understand how the achievement levels were applied. It becomes even more difficult for moderators when the general summary remarks are either not given or consist of one sentence comments taken verbatim from the published achievement level descriptors. Teachers should remember that the main moderator’s responsibility is to confirm the teacher marks, and this can only be done effectively with the help of specific annotations and summary comments. As stated in the TSM at the end of the process teachers should verify the accuracy of all calculations and annotate the work appropriately to indicate where and how achievement levels have been awarded.

Many explorations were seen that only paraphrased work from literature giving the students no room for personal engagement or the use of their own mathematics. It continues to be an issue that teachers seem to encourage research reports and award higher levels in all criteria and often impressed with the level of sophistication and rigour in the mathematical content. Some bibliographies were not checked, and it was noted that some students chose not to include some sources used. When there is suspicion of this happening, moderators will raise exceptions to indicate referencing irregularities. Some sources could be identified by the moderators as cheat sites which specifically identify work as “IB Internal Assessment”. With reference to the use of AI, although this is generally allowed, it should be acknowledged as a source by the student. This only happened a handful of times. Several bibliographies were several pages long. Students should be advised to only include sources that were explicitly used in the Exploration in their bibliography. Many explorations involved volumes and surface area of revolution and several poorly written statistical explorations were moderated, that did not meet the expectations for higher achievement levels.

Teachers and students need to understand that the target audience is a peer and that the work needs to be accessible to a peer without having to rely on any outside sources. When the mathematics used is not part of the syllabus content, this becomes more difficult to achieve within the page limit. It continues to be a concern that common topics such as the Fourier Series and/or transforms, SIR model and The Golden Ratio are still being submitted. Although it is true that for a student these are interesting topics, a research report in which a candidate simply transcribes work that is easily found online or in textbooks makes higher achievement levels unattainable.

There was evidence to suggest that some teachers did not offer sufficient and appropriate guidance to students during the developmental stage of their internal assessment. One of the responsibilities of the teacher is to provide students with feedback on work completed at various stages of the exploration and to give assistance to individual students in overcoming particular problems.

It is acknowledged that some interesting explorations were submitted which clearly demonstrated personal engagement and student connection to the mathematical processes used in the topics chosen.

Student performance against each criterion

Criterion A

High achievement levels in this criterion should be accessible to most students. Although the quality of written work was good, several students miss the higher levels for various reasons, including long explanations that are unrelated to the aim, repetition, and irrelevant extensions. It was again noted that several students wrote out a method/plan of what was to follow in the introductory page. An overview of the work may help students to ensure that the exploration remains focused, but it is not necessary to submit the plan within the body of work. When the topic is too ambitious, the work either becomes too long or incoherent due to lack of sufficient clarification. Other times students do not link back to the original aim given in the introduction which leads to sections which are linked to the topic but irrelevant to the aim. Some strategies used to maintain conciseness included placing a good amount of work in an appendix or tabulating the mathematics within the work. Although a page limit is recommended, it is not the actual number of pages that determines whether the work is concise. It was also noted that several students provided an "Evaluation" after the conclusion, which has a negative impact on organization.

Criterion B

In general students scored well in this criterion, however, the top achievement level proved difficult to award because of several reasons. Careless mistakes included unlabelled diagrams, tables split over two pages, subtitles at the bottom of a page as well as the use of computer notation, particularly the use of "*" for multiplication and the misuse of "=" sign when the "≈" sign was required. It should also be noted that key terms need to be explained when the work refers to subject matter outside of the syllabus content, including reference to terms from other disciplines. However, there is no need to explain any topics within the syllabus content in detail. With teachers giving written feedback on drafts of the work, all students should be able to achieve the highest level in this criterion.

Criterion C

The descriptors for achievement levels in this criterion continue to be misinterpreted by teachers. Neither a statement by the student that over enthusiastically explains how interested they are in the topic, nor statements made by the teacher about the effort that went into completing the IA contribute towards this criterion. There are several ways in which a student can personalize a topic including the collection of data. However, this is not enough to demonstrate authentic personal engagement unless this data is used effectively in mathematical processes which allow the topic in question to be addressed from different perspectives. With that said, it is often observed that when modelling is used, students will include irrelevant models erroneously thinking that this will present varied perspectives. Work that is heavily reliant on published sources will not allow for the highest achievement to be awarded. Students who present explorations based on common textbook problems, historical reports, or the derivation of a formula, e.g., surface area of revolution, are restricted from taking ownership of the work and showing creativity.

Criterion D

The expectation in this criterion is that students pause and reflect about their work as the exploration is being developed. Unfortunately, some teachers interpret this as giving descriptive explanations before moving on. Ongoing reflection should have a candidate reviewing results and evaluating the appropriateness mathematical processes and/or strategies used. The highest achievement level should not be elusive for students if they refrain from visual interpretation of graphical models, and descriptions of results. It is worth repeating a statement mentioned in previous subject reports: Good reflection will include the candidate asking themselves questions like: “How reliable is this result?”, “What if…?” or “How can the process be changed to improve reliability?” The answer to such questions followed up by action will invariably result in meaningful and critical reflection. Although mentioned in previous subject reports, the addition of “limitations and extensions” continues to be seen in many explorations. This inclusion, as an afterthought contributes minimally towards this criterion, and it should be mentioned that all topics lend themselves to such a discussion. It follows that choosing a topic that is way beyond the syllabus content, or one that simply paraphrases published results will not allow the candidate to reflect effectively.

Criterion E

In general students used relevant mathematics commensurate with the level of the course but few students managed to achieve the higher levels. The misconception that the mathematics used needs to be included in the “Additional HL” topics in the syllabus for it to be commensurate with the level of the course continues to be observed. Anything that is within the syllabus content is commensurate with the level of the course, however if the use of mathematics is not beyond what would be expected of an SL candidate, the higher achievement levels cannot be awarded. Although less frequent, some teachers continue to encourage students to use mathematics that is well beyond the level of the course with the result that students cannot explain the mathematics used. This issue also occurs when the topic is closely related to Physics, resulting in students having to use mathematical processes that are too difficult. At the other end of the spectrum, statistical explorations are often seen where students use correlation without showing understanding that this does not imply causality. If a model is to be obtained from the collected data, some form of testing needs to be done to test dependence.

Recommendations and guidance for the teaching of future students

It is of utmost importance that teachers read the TSM pages related to Internal assessment and the Subject Reports to understand the objectives of this component. Teachers are expected to guide students and highlight opportunities about topics that might lend themselves for the IA as the course is being taught.

Although the topic is chosen by the student, a topic is proposed that does not lend itself to an Exploration, it is within the teacher’s remit to refuse it and to guide the student towards a more appropriate topic. Some teachers still fail to interpret the different criterion descriptors correctly which results in marks being adjusted in moderation. There are 49 marked exemplars on the Programme Resource Centre. These have now also been organized into a spreadsheet making it easier for teachers to choose which explorations to read and share with their students.

Some students are left to their own devices after submitting their topic / subject proposal. This is evident when the work submitted could have attained higher levels with some more guidance.

Whenever possible the exploration should not be finalized too early in the course as this often precludes the students from achieving higher levels. Unless the students can obtain guidance on mathematics that has not yet been taught, they will often miss out on the sophistication and rigour of the mathematics that is required. Students should be advised to cite sources at the point of reference and not just in the bibliography. The teacher should also check these citations and the bibliography to ensure that there is no risk of exceptions being raised for plagiarism or reference irregularities by moderators. Teachers and students should read the document “Academic Integrity” published in 2019 and updated in March 2023, which is available on the Programme Resource Centre.

Further comments

Some of the comments below have been mentioned in previous reports but it is important to reiterate them here.

Far too many uploaded explorations were clean, without any teacher annotations. Such annotations are very helpful to moderators who strive to verify the teacher marks.

Schools should have a system in place that emphasizes academic integrity. One of the ways of ensuring authenticity is to provide teachers with a platform where they can supervise the development of each IA remotely and with access to communicate with individual students.

Higher level paper one

General comments

It was encouraging to see the students' progress this session particularly their response to section B questions and the Higher-Level syllabus topics. Most students were able to present their working in a clear and logical manner. However, more practise is needed in topics such as complex numbers and proof by induction. Another area for attention could be to improve the quality of graph sketches.

The areas of the programme and examination which appeared difficult for the students

- Finding sum of a geometric series.
- Proof by mathematical induction.
- Maclaurin series expansion using simple substitution and use of products of trigonometric functions.
- Sum and product of roots of a cubic function.
- Complex numbers.

The areas of the programme and examination in which students appeared well prepared

- Mean of grouped data.
- Logarithms.
- Quadratic functions.
- Factor theorem and factorising a cubic into linear factors.
- Sketching graphs of rational and absolute value functions.
- Finding partial fractions with two distinct linear factors.

The strengths and weaknesses of the students in the treatment of individual questions

Question 1

This question was generally well done by vast majority of the students, although more than a few could not get the second equation in part (a).

Question 2

Part (a) was well done using laws of logarithms and rational exponents with many completely correct answers seen. Part (b) was relatively less successful compared to part (a), a number of students did not recognise the need to use change of base rule, of those who did a few made arithmetic errors.

Question 3

In Part (a), those who set-up two correct equations were often successful in obtaining the required quadratic equation. Some students incorrectly used arc length formula for perimeter of the sector leading to invalid results, a significant number recognised only one equation which was frequently seen in part (b), a few failed to eliminate θ. Part (b) was well attempted with many students showing good progress.

Question 4

Part (a) was well done. Part (b) was well attempted with many completely correct answers seen. Some accuracy errors were noted in the integration of $\sin 2x$ or with the application of limits and the calculations thereafter.

Question 5

Part (a) was answered correctly by almost all students in this paper. Part (b) proved to be challenging for most students with only a few correct answers seen.

Question 6

Many students set up the correct integral for finding the volume of revolution and a good number of completely correct answers were seen using inspection or substitution. A significant number of students used integration by parts instead of substitution and lost their way in this problem.

Question 7

This question proved to be challenging for majority of the students with only a few correct answers seen. Although students knew the format for proof by induction, many had difficulty in executing these procedures correctly in this problem losing marks at various stages. Common errors include failure to make assumption of truth clear at the assumption step, not stating the value for the base case, difficulty working with the fractions involving factorials and sigma notations.

Question 8

In part (a) (i), many students correctly used the Maclaurin series for $\sin x$ given in the formula booklet to obtain the series for $\sin\left(x^2\right)$ as required. In part (a)(ii), a surprisingly large number of students chose to square each term individually rather than the entire series. Many students did not attempt part (b) and the response was generally poor for this part with very few correct answers seen.

Question 9

This question was well attempted with many completely correct answers to part (a) and (b). There was a mixed response to part (c) and some did not attempt this part.

Question 10

This question was generally well done. The sketch of a rational function in part (a) was well attempted with many gaining more than half of the allocated marks for this part. There were many excellent responses to parts (b) to (e) showcasing a variety of approaches involved in solving the quadratics. Although some progress was seen in part (f), some made arithmetic errors in finding the cubic and only a very few recognised the need to use the product of the roots thereby obtaining the correct final answer.

Question 11

Parts (a), (b) and (c) were well done with many completely correct answers seen. A good number of students did not attempt the remaining parts in this question. Part (d) proved tricky with many struggling to show the required result. Those who attempted parts (e) and (f) made some progress with stronger students gaining full marks. Some did not recognise the need to use L'Hôpital's rule in part (f).

Question 12

Parts (a) and (b) were well attempted with more than a few students making arithmetic errors in the binomial expansion involving complex numbers. In part (a)(ii), many students seem to be unaware that the imaginary part is stated without the i. In part (b), students often used only the real part from part (a) to incorrectly arrive at the result which resulted in loss of accuracy marks. Many reversed v and w in part (c), some did not know how to find the third root. Reasonable attempts at parts (d) and (e) were seen although many did not get this far. A few gave the area in terms of a complex number in part (c) which is invalid. Some did not recognise the requirement in (e) for the arguments to be in the given domain. Parts (f) and (g) were poorly attempted with some gaining Follow Through for part (f). Part (g) proved challenging to almost everyone with only a handful of correct answers seen.

Recommendations and guidance for the teaching of future students

- Emphasize the importance of writing all essential working in a legible manner.
- When sketching curves, encourage the importance of drawing smooth curves, make sure students include suitable scales and label their graphs.
- Students need to be familiar with the contents of the formula booklet and make effective use of the same by encouraging them to work with this in their assignments.
- Emphasize the importance of the procedure to follow in 'show that' questions. Working backwards is not an appropriate method.
- Encourage students to practise formal setting out of proofs throughout the course. More practise with different types of proof by induction with emphasis on correct notation.
- Encourage use of Argand diagrams to represent complex numbers. Ensure that students can work with complex numbers in all three forms. Provide opportunities to investigate transformations involving complex numbers.

Higher level paper two

General comments

This paper was often well done, with continued evidence of improving calculator use, especially in questions involving kinematics or graphing. A few students lost a lot of marks for repeatedly giving answers to 2 significant figures; this was seen most often in probability questions. It is also worth noting that students need to pay attention to the logical presentation of their solutions so that their method is clear to examiners.

The areas of the programme and examination which appeared difficult for the students

Students struggled especially to present a clear strategy to solve complex combinatorics problems, and also to explore a 3-dimensional geometrical situation involving vectors. They found it difficult to work with the graph of a relation defined implicitly, including finding the coordinates of points using properties given in the question. Many students could not find the median of a continuous random variable whose probability density function was given by a piecewise function, by considering the nature of the function and using a calculator efficiently to find the limit of an integral. Recognising the need to use an integrating factor to solve a non-separable, non-homogeneous differential equation was also a problem.

The areas of the programme and examination in which students appeared well prepared

Students performed well on most of the Standard Level topics, and it was pleasing to see more effective calculator use this year. Problems involving depreciation, trigonometry and volume were usually solved successfully, as were problems involving analysis of a given sinusoidal model. Of the Higher Level topics, the best answered questions involved implicit differentiation and manual integration of a continuous probability density function to determine a constant.

The strengths and weaknesses of the students in the treatment of individual questions

Question 1

Parts (a) and (b) were usually well answered, although some students misinterpreted the question and worked with percentage increase rather than decrease. A number of students lost a mark in part (b) through not rounding to the nearest dollar.

Part (c) was well answered by the majority of students, the most common error being to forget to add the initial year back on to obtain the total number of years.

Question 2

Students usually gained at least one mark in part (a); many scored full marks. A common mistake was to use a height of 20m rather than 18.2m in the trigonometry calculation.

Part (b) was well done, with many students gaining Follow Through marks from an incorrect radius.

Question 3

This question caused some difficulty for candidates who struggled to interpret and visualise the problem; very few drew a diagram to support their thinking. Many answered part (a) correctly, but struggled to score full marks on part (b) by equating an expression in k to a value obtained using an inverse normal calculation.

Some students recognised that both question parts could be solved by considering the standard normal distribution, reflecting excellent understanding.

Question 4

Good use was made of the calculator in this question, especially in parts (a) and (c) which were well done, although some students still attempted unnecessary algebraic manipulation. In part (b) it was common for students not to understand that increasing displacement meant positive velocity, and some also lost marks for incorrect rounding or poor notation.

Question 5

This was the most poorly answered of the Standard Level questions, with many students misunderstanding the appropriate use of regression lines. A common response to part (b) was to say that Paulo is an outlier; whether this was meant in the context of univariate or bivariate statistics was not clear. The notion of his score being outside the range of scores used to derive the regression line and the dangers of extrapolation were often not clearly expressed.

Question 6

Parts (a) and (b) were well done by those who recognised a binomial distribution, but this was not as common as expected. The most common mistake was to use an incorrect approach $\mathrm{P}(X \geq 10) = 1 - \mathrm{P}(X \leq 10)$.

Many candidates struggled to answer part (c), which relied on a simple product of independent events.

Throughout this question especially, a number of candidates lost multiple marks for giving their answers to only 2 significant figures.

Question 7

This question proved to be challenging for the students. Many tried to solve this as a homogeneous differential equation and did not make any progress. Those who did recognise that an integrating factor was required generally scored full marks, with a good understating of how to use integration by parts.

Question 8

Part (a) was generally clear and well done.

Students struggled to answer part (b), missing the critical point that the median lay within the second interval in the domain of the piecewise function. Some students made good progress towards an integral, but appeared unable to use their calculator to find the required limit and instead attempted an algebraic approach.

Question 9

Part (a) was often well done, although some students did not multiply by 2, taking into account the choices of position for Alvin and Bobby.

Part (b) was poorly done. It was difficult to award partial marks because it was often very hard to understand the strategy of the candidate – a clear consideration of different cases is essential when presenting solutions to problems of this nature.

Question 10

Parts (a), (b) and (c) were well done by most students. Some premature rounding led to an incorrect value in part (a). It was common to see good use of the calculator, although some still spent unnecessary time on algebraic work, especially in part (c) where many differentiated by hand.

Part (d) was less successful. Many students were able to find the values of a and d, although some of them assigned them the wrong way round and thus lost marks. It was less common to see a good strategy to find the value of b, and even more unusual to see a strategy for finding the value of c. A common error was to use half of the period rather than the full period. Some students successfully solved the problem using simultaneous equations in b and c.

Part (e) was well done by students who had arrived at a set of values in part (d), with many gaining Follow Through marks.

Question 11

Part (a) was well done by most students. A small number tried to work backwards from the given answer so not all the steps followed from each other.

Parts (b), (c) and (d) proved challenging, although a good number recognised that part (b)(ii) was accessible even to those who struggled with part (b)(i).

Question 12

Most students completed part (a) successfully, although it was disappointing to see such a simple identity proof set out so poorly by so many who clearly had the right ingredients to hand.

Part (b) required the assembly of many pieces of a jigsaw, and many students successfully put a number of them together. The final step required the use of the scalar product $\boldsymbol{u}\cdot\boldsymbol{v}$ and the modulus of $\boldsymbol{u}$; very few managed to derive a correct quadratic equation in one variable.

Part (c) was out of the reach of most students, and it was rare to see a good representation of the problem. Many attempted to find the coordinates of the point D rather than focusing on the vector $\boldsymbol{w}$, making the solution more challenging.

Recommendations and guidance for the teaching of future students

Students should be made aware of the requirements of giving rounded answers exactly or to 3 significant figures, unless specifically asked to do otherwise in the question.

It is important that students know how to break down a multi-step problem and identify key information given in the question.

Students need to set their work out clearly and logically, rather than cramming their work into a small space, making it difficult to follow.

It would be useful for students to know how to use their calculator to find the limit of an integral where the value of the integral is known.

Students need to know how to prove identities logically, usually from left to right or vice versa, including each key step and not leaving it to the reader to make assumptions.

The validity of regression lines should be taught alongside the mechanical process of finding them.

Higher level paper three

The areas of the programme and examination which appeared difficult for the students

- Reasoning; Many students could not respond adequately to questions that required logical explanations.
- Writing all the necessary steps needed for a "Show that" command
- Differentiation of a composite function
- Problem solving with an unfamiliar probability model
- Calculating the variance of discrete random variables
- Factoring polynomials that have parameters for coefficients

The areas of the programme and examination in which students appeared well prepared

- Probability; such as tree diagrams and conditional probability
- Differentiating polynomials
- Finding points of inflexion
- Finding the gradient/slope of a line
- Finding points of intersection using the GDC

The strengths and weaknesses of the students in the treatment of individual questions

Question 1

- (a)(i) Generally well answered. A common error was trying to use the binomial distribution.
- (a)(ii) Generally well answered. Use of binomial distribution was the most common method. Students could correctly identify the parameters for the distribution, but common errors were finding $\mathrm{P}(A>1)$ instead of $\mathrm{P}(A\geq 1)$ or finding $\mathrm{P}(A<1)$ or finding $\mathrm{P}(A=1)$. This could be due to not using the GDC correctly when calculating a probability for a binomial distribution.
- (b)(i) Reasonable response, but some students did not make clear the order in which the actions occur. Some students gave lengthy responses, which is not needed for a 1 mark question.
- (b)(ii) Many students struggled to make the connection between the formula for $\mathrm{E}(X)$ given in the formula book and the probability given in part (i). Common errors included $\mathrm{E}(X)=\sum_1^\infty p(1-p)^{x-1}$ or $\mathrm{E}(X)=np$ or simply writing the formula for the expected value of a discrete random variable from the formula booklet, without using the probability given in part (i). Some students wrote incorrect limits, but this was condoned during marking.
- (c)(i) Some students made errors in the differentiation, which was surprising given the emphasis on calculus in the Analysis and Approaches course. Perhaps the use of variable r instead of the more familiar x created difficulty for some students. The most common error was a negative sign on the RHS, but there were also errors differentiating the LHS. Some students could do the differentiation correctly,

but could not make the connection between their answers and the summation asked for in the question.

- (ii) Due to the errors in (b)(ii) and (c)(i), only the strongest students could make progress in this question.
- (d) The majority of students could find the variance correctly using the given formula. It was surprising that many students could not find $\mathrm{E}(X)$, using incorrect methods such as $\mathrm{E}(X)=np$, instead of using the formula given in (c)(ii).
- (e) A well answered question, with many students demonstrating a good understanding of conditional probability.
- (f) Many students gave imprecise reasoning, assuming that the probability of a boost always increases by 0.2 and not making clear that this only occurs when the action has not been boosted. A common misconception was thinking that the probability equals one after five actions, instead of after four actions that have not been boosted.
- (g)(i) Reasonably well answered. Many students recognized the need to multiply probabilities to find the value of m and n.
- (ii) Reasonably well answered, most students made the connection to the formula in the formula booklet.
- (iii) Not well answered. Despite the formula being in the formula booklet, many students could not use it correctly, e.g. squaring the probabilities instead of squaring the y values. Some students got a negative answer, and did not seem to realize this is impossible. Some students used their GDC, which was an efficient method.
- (h)(i) Some students did not respond to the final parts of the question, but those that attempted this part generally did well.
- (ii) Some students used incorrect formulas for the variance, such as $np(1-p)$ instead of using the formula given before part (d).
- (iii) Students who had calculated the variance of X and Y generally scored well here, recognizing that a lower variance would lead to a more consistent experience for the player.

Question 2

- (a) Well answered, many students could find the first and second derivative of a polynomial. A small number of students did not seem familiar with the notation used in the question.
- (b) Well answered, many students understood how to find points of inflexion. There was evidence of many students making effective use of their GDC to help.
- (c) Most students could use their answers from part (b) to show the gradient is -0.375, but some lost a mark for not giving a reason why the y-intercept is zero. Also, some solved backwards, checking the solution, which was only awarded partial credit
- (d) well answered, although some students lost a mark for not giving 3 decimal places. This could be due to GDC settings and unfamiliarity with adjusting them.
- (e) Reasonably well answered, although some weaker students struggled with the algebraic demands, even though they had been successful in part (a). The inclusion of variables m and n made the question more challenging for these students.
- (f)(i) (ii) Well answered. Some students made algebraic errors, such as incorrectly expanding $m\left(\frac{m}{2}\right)^3$, which was unexpected in a HL exam.

- (g) Fine, although some students did not understand that all algebraic steps must be written for a "Show that" command.
- (h) This question was designed to be a challenge and many students struggled to make any progress beyond simplifying the equation equal to zero. A few then noticed the common factor of x but most of those divided by x without justification. Successful students made the connection to the roots found in part (e), with the most common approach being use of the factor theorem and long division. Some students used the formulas for the sum and product of the roots. It was pleasing to see a significant minority of students scoring well in this question
- (i) A number of students could use the given result in part (h) successfully in part (i). A common error was using y_C in the denominator instead of x_C. Another common error was making a sign error when simplifying $0-\left(\frac{m}{4}-\frac{m}{4}\sqrt{5}\right)$. This could be due to students rushing to complete the question at the end of the exam. Interested teachers might like to highlight to their students that this ratio is the reciprocal of the golden ratio.

Recommendations and guidance for the teaching of future students

- Ensure that students understand the command words. In particular, "Show that" means that all necessary algebraic steps should be shown. This is particularly relevant in paper 3, where "show that" is used to help students progress through a question, in case they get stuck on one part. Some students appeared to know what to do, but skipped key parts of their steps, perhaps assuming they were "obvious". However, students need to convince the examiner they know what they are doing. Teachers should emphasize that when it comes to showing the validity of a given result, students cannot work backwards from the given result, but should derive it (often using previous parts).
- Enable students to be guided by the mark allocation for each part. E.g. a 1 mark explanation should only require 1 or 2 sentences to answer. Allow students to practise this and have them peer edit, to further develop their skills to communicate clearly and concisely.
- Emphasise that exact numbers and numbers to more than 3sf are generally more useful than rounded ones when using them in subsequent calculations.
- Insist on the use of correct terminology e.g. there is a difference between "the x-coordinate of" and "the coordinates of".
- Ensure students are familiar with the notation used in the guide.
- Ensure that all of the syllabus is taught. Some students scored highly in one question, but left much of the other question blank.
- Practise factorising polynomials, particularly in harder contexts, such as when the coefficients are parameters.
- It is good to emphasize to students that they look at the question as a whole, so that they can better understand which results are given and which results are intended to be determined.
- Students should be advised only to cross out their work, if they replace it with new work.
- Some students handwriting is difficult to read, please encourage students to set out their working clearly.

Further comments

Time management for this paper appeared to be better than some previous papers, but there was still evidence of some students running out of time.